MATH-S400

Assignment 1: logic and proofs

1. Three mathematicians (that take things very literally) walk into a bar. The waiter asks: "Do you all three want a drink?" The first mathematician says: "I don't know." The second mathematician says: "I also don't know." The third mathematician says: "Yes".
   Explain the reasoning of each of the mathematicians.

2. Write down the truth table to derive the validity of the expression $q \to (p \to q)$. Also give the derivation without truth tables (write down the formula using only $\neg, \wedge$ and $\vee$).

3. Negate the following expressions (for $A \subseteq \mathbb{R}^n$)

   - $\forall \varepsilon > 0, \exists \delta > 0, \forall x, y \in \mathbb{R} : \|x - y\| < \delta \to \|f(x) - f(y)\| < \varepsilon.$
   - $\forall \mathbf{x} \in A, \exists \varepsilon > 0, \forall \mathbf{y} \in \mathbb{R}^k : \|\mathbf{x} - \mathbf{y}\| < \varepsilon \to \mathbf{y} \in A.$
   - $p \to (q \leftrightarrow z).$

   For the last one, express the negation such that it only uses the following symbols: $\neg, \wedge$ and $\vee$.

4. Show that for any list of $n + 1$ numbers $a_1, \ldots, a_{n+1}$ in $\mathbb{N}$, I can find at least two numbers in this list whose difference is a multiple of n. For example if $n = 3$ then a list might be $7, 8, 13, 21$ here I have that $6(= 13 - 7)$ is dividable by 3. (Hint: Show first that at least two numbers have the same remainder after dividing by n.)

5. Proof that for all numbers $x \in \mathbb{Z}$ and all $n \in \mathbb{N}$: x^n is odd if and only if x is odd.