MATH-S400
Assignment 3

1. Let $S \subseteq \mathbb{R}^n$ be a compact set and let $C : S \twoheadrightarrow S$ be an upper hemi-continuous correspondence. Consider the correspondence $D : S \twoheadrightarrow S$ where,

$$D(\mathbf{x}) = \{\mathbf{y} \in S | \exists \mathbf{z} \in S, \mathbf{z} \in C(\mathbf{x}) \text{ and } \mathbf{y} \in C(\mathbf{z})\}.$$

   D is the two-fold composition of C. Show that the correspondence D is also upper hemi-continuous.

2. Let $G : \mathbb{R}_+ \twoheadrightarrow \mathbb{R}_+$ be given by,

$$G(x) = \{y \in \mathbb{R}_+ | 0 \leq y \leq x\}.$$

   Show that G is upper and lower hemi-continuous.

3. Let $T = \mathbb{R}$, $G(\theta) = [-1, 1/2]$ and $f(x, \theta) = \max\{x - \theta; \theta - x\}$. Consider the problem,

$$\max_x f(x, \theta) \text{ s.t } x \in G(\theta).$$

   Are the assumptions of Berge's maximization theorem satisfied. Solve this problem and graph the optimal value function and the solution correspondence. Is this correspondence upper hemi-continuous, lower hemi-continuous? why/why not?

4. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined as,

$$f(\mathbf{x}) = f\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = |x_1 - x_2|.$$

   Is this function convex? why/why not?