MATH-S400

Assignment 1: logic and proofs

1. Assume that there are six cards on the table in front of you. Each card has a number on one side and a letter on the other side. On the cards, you can see $A, 4, 6, L, E$, and 7. If I claim that when a card has a vowel on one side, then it has an even number on the other. How many cards do you have to turn over minimally to check this? Which cards are those?

2. Write down the truth table to derive the validity of the expression $(p \to q) \to q$. Can you do it without truth tables.

3. Negate the following expressions (for $A \subseteq \mathbb{R}^n$)

   - $\forall \mathbf{x} \in A, \forall \varepsilon > 0, \exists \delta > 0, \forall \mathbf{y} \in A : \|\mathbf{x} - \mathbf{y}\| < \delta \to \|f(\mathbf{x}) - f(\mathbf{y})\| < \varepsilon.$
   - $\forall x, y \in \mathbb{R}, \exists z \in \mathbb{Q} : (x < y) \to (x < z < y).$
   - $\forall x \in \mathbb{R}, \exists n \in \mathbb{N} : x < 1/n.$

4. For a set $C \subseteq \mathbb{R}$ let $C^c = \mathbb{R} \setminus C$. Show that for any two sets A and $B \subseteq \mathbb{R}$,

$$[(B \cup A^c)^c \cup B] = A \cup B.$$

5. Prove that for all $n \in \mathbb{N}$:
   For all numbers $x_1, \ldots, x_n \in \mathbb{Z}$: the product

$$x_1 \cdot x_1 \ldots x_{n-1} \cdot x_n,$$

   is even if and only if for at least one $i \leq n$, x_i is even.