Overview of some important distributions

X is a continuous random variable

Notation	*Parameters*	$x \in S$	*Pdf*	$E(X)$	$Var(X)$
$N(\mu, \sigma^2)$	$\mu \in \mathbb{R}, \sigma \in \mathbb{R}_0^+$	$x \in \mathbb{R}$	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$	μ	σ^2
$N(0,1)$	/	$x \in \mathbb{R}$	$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$	0	1
$Exp(\lambda)$	$\lambda \in \mathbb{R}_0^+$	$x \in \mathbb{R}_0^+$	$\lambda e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
$U(a,b)$	$a, b \in \mathbb{R}$	$x \in [a,b]$	$\frac{1}{b-a}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
χ_r^2	$r \in \mathbb{R}_0^+$	$x \in \mathbb{R}_0^+$	$\frac{2^{-r/2}}{\Gamma(r/2)} x^{\frac{r}{2}-1} e^{-\frac{x}{2}}$	r	$2r$
t_r	$r \in \mathbb{R}_0^+$	$x \in \mathbb{R}$	$\frac{\Gamma((r+1)/2)}{\Gamma(r/2)\sqrt{\pi r}} (1 + \frac{x^2}{r})^{-\frac{r+1}{2}}$	0	$\frac{r}{r-2}$
$F_{n,m}$	$n, m \in \mathbb{R}_0^+$	$x \in \mathbb{R}_0^+$	$\frac{\Gamma((n+m)/2)}{\Gamma(n/2)\Gamma(m/2)} n^{\frac{n}{2}} m^{\frac{m}{2}} \frac{x^{\frac{n}{2}-1}}{(m+nx)^{\frac{n+m}{2}}}$	$\frac{m}{m-2}$	$\frac{2m^2(m+n-2)}{n(m-2)^2(m-4)}$

Some explanation:

- $N(\mu, \sigma^2) =$ the normal distribution
  - Family of symmetric distributions
  - Used a lot because of central limit theorem
  - Standard normal distribution if $\mu = 0$ and $\sigma = 1$
- $Exp(\lambda) =$ the exponential distribution
  - Right skewed distribution
  - Life expectancy of objects (machines, humans, ...)
- $U(a,b) =$ the uniform distribution

- Pick a random number between a and b

- χ^2_r = the chi-squared distribution with r degrees of freedom
  - Used in statistical and econometrics test
  - Let $X_1, \ldots X_r \sim N(0,1)$ and independent from each other, then $Z(= \sum_{i=1}^{r} X_i^2) \sim \chi^2_r$
  - $\Gamma(t) = \int_0^\infty x^{t-1} e^{-x} dx$ is the Gamma function
  - Right skewed distribution

- t_r = the t-distribution or student distribution with r degrees of freedom
  - Used in statistical and econometrics test
  - Let $X_1 \sim N(0,1)$, $X_2 \sim \chi^2_r$ and X_1 and X_2 be independent, then $Z(= \frac{X_1}{\sqrt{X_2/r}}) \sim t_r$
  - Similar to the normal distribution but heavier tails
  - In the limit the same as the normal distribution

- $F_{n,m}$ = the F-distribution with n and m degrees of freedom
  - Used in statistical and econometrics test
  - Let $X_1 \sim \chi^2_n$, $X_2 \sim \chi^2_m$ and X_1 and X_2 be independent, then $Z(= \frac{X_1/n}{X_2/m}) \sim F_{n,m}$
  - Right skewed distribution

X is a discrete random variable

Notation	*Parameters*	$x \in S$	$Prob(X = x)$	$E(X)$	$Var(X)$
$B(1, p)$	$p \in]0, 1[$	$x \in \{0, 1\}$	$p^x(1-p)^{1-x}$	p	$p(1-p)$
$B(n, p)$	$p \in]0, 1[, n \in \mathbb{N}$	$x \in \{0, \ldots, n\}$	$\begin{pmatrix} n \\ k \end{pmatrix} p^x(1-p)^{n-x}$	np	$np(1-p)$
$Poisson(\lambda)$	$\lambda \in \mathbb{R}_0^+$	$x \in \mathbb{N}$	$\frac{e^{-\lambda}\lambda^x}{x!}$	λ	λ
$U(N)$	$N \in \mathbb{N}_0$	$x \in \{1, \ldots, N\}$	$\frac{1}{N}$	$\frac{1+N}{2}$	$\frac{N^2-1}{12}$

Some explanation:

- $B(1, p)$ = the Bernoulli distribution
  - Only two possible outcomes
  - E.g. 1 = success, 0 = fail
  - p is the probability for success
  - E.g. flip a coin
- $B(n, p)$ = the binomial distribution
  - n repetitions of a Bernoulli experiment with probability of success equal to p
  - $Prob(X = x)$= what is the probability of having x times a 1 in the n repetitions
  - E.g. flip a coin n times, what is the probability of having x tails

- $Poisson(\lambda)$ = the Poisson distribution
  - The number of events in a given time frame
  - Is the limiting distribution of the binomial distribution (i.e. the number of repetitions gets very big and the probability of success gets very small)
  - E.g. the number of telephone calls per day
- $U(N)$ = the uniform distribution
  - The discrete counterpart of the uniform distribution above